Estimation of Pickands dependence function of bivariate extremes under mixing conditions

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Modeling dependence structures of multivariate extremes is of great interest in many application fields . A well known way to model these structures is to use Pickands dependence function (Pickands, 1981). If $X = (X_1, X_2)$ is a bivariate vector of extremes with margins F_1 and F_2, Pickands dependence function A is defined via the extreme-value copula's type representation:

$$C(u,v) = \mathbb{P}(F_1(X_1) \leq u, F_2(X_2) \leq v) = \exp\left\{\log(uv)A\left(\frac{\log(u)}{\log(uv)}\right)\right\},\ 0 \leq u, v \leq 1,$$

and totally characterizes the joint distribution $F(x_1, x_2) = C(F_1(x_1), F_2(x_2))$ of (X_1, X_2) of X knowing its marginal laws. It may be shown that $A : [0,1] \to [1/2, 1]$ is a convex function such that $A(0) = A(1) = 1$ and $\max(t, 1-t) \leq A(t) \leq 1$.

The problem of estimating Pickands dependence function by nonparametric methods has been extensively studied in the literature (see Zang et al., 2008). The underlying sequence of extremes observations is always assumed to be i.i.d., which excludes a possible serial correlation. This bias is to a certain extent supported by theoretical results on maxima of strictly stationary sequences (see e.g. Hsing, 1989). In practical situation however, the temporal independence of extremes is usually an unrealistic assumption. In the sequel, we propose to revisit the properties of the so called CFG estimator, a classical estimator of A (see Capéràa et al., 1997), when it is based on some kind of weakly dependent strictly stationary sequence of extremes, then to use these properties to build a test of independence of X's margins.